Circular Motion and Gravitation

I – Kinematics of Circular Motion

Circular Motion

- An object moves in a straight line if the net force on it acts in the direction of motion, or is zero.
- If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path.

Kinematics of Uniform Circular Motion

- An object that moves in a circle at constant speed, **v**, is said to experience **uniform circular motion.**
  - The *magnitude* of the velocity remains constant, but the *direction* of the velocity is continuously changing .
  - Since acceleration is defined as the rate of change in velocity, a change in *direction* of v constitutes an acceleration just as does a change in magnitude

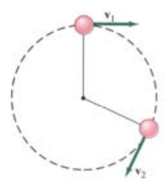


FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. Note that at each point, the instantaneous velocity is in a direction tangent to the circular path.

Kinematics of Uniform Circular Motion

- SO, an object revolving in a circle is continuously accelerating, even when the speed remains constant.
- Acceleration is defined as:

$$a = \frac{v_2 - v_1}{\Delta t} = \frac{\Delta v}{\Delta t}$$

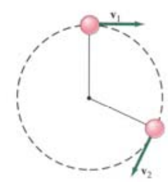
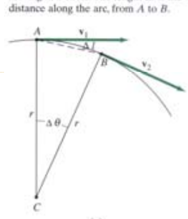


FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. Note that at each point, the instantaneous velocity is in a direction tangent to the circular path.

Kinematics of Uniform Circular Motion

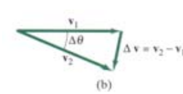
- During Δt, the particle moves from point A to point B, covering a distance Δl *along the arc* which defines an angle $\Delta\theta$.
- Let Δt, be VERY small. Then Δl and $\Delta\theta$ are also very small and v_2 will be almost parallel to v_1 and Δv will be essentially perpendicular to them

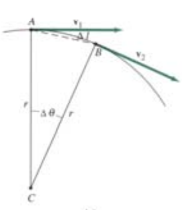
FIGURE 5–2 Determining the change in velocity, $\Delta \mathbf{v}$, for a particle moving in a circle. The length Δl is the distance along the arc, from A to B.



Kinematics of Uniform Circular Motion

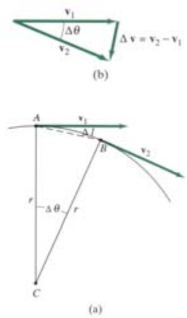
- The Δv vector points inward toward the center of the circle and **a**, by definition above, is in the same direction as Δv, it too must point toward the center of the circle and **centripetal acceleration** (center-seeking) is born!
- It is also known as radial acceleration, a_R since it is directed along a radius toward the center of the circle.





Kinematics of Uniform Circular Motion

- For a VERY small $\Delta\theta$
- $\frac{\Delta v}{v} = \frac{\Delta \ell}{r}$ for $v_1 = v_2 = v$
- $\Delta v = \frac{v}{r}\Delta \ell$
- To get a_R, we divide Δv by Δt
- $$a_R = \frac{\Delta v}{\Delta t} = \frac{v \Delta \ell}{r \Delta t} = \frac{v^2}{r}$$



Summary

- An object moving in a circle of radius r with constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is v^2/r.

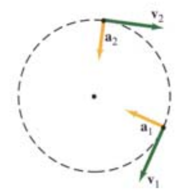


FIGURE 5–3 For uniform circular motion, **a** is always perpendicular to **v**.

Definitions

- Frequency – f, the number of revolutions per second.
- Period – T, time required for one complete revolution. Equal to $1/f$
- For an object revolving in a circle with constant speed:

$$v = \frac{2\pi r}{T}$$